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10CS42

Fourth Semester B.E. Degree Examination, June/July 2016
Graph Theory & Combinatorics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

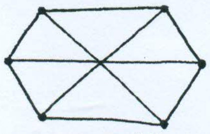
PART – A

1 a. Define : i) Complete Bipartite Graph ii) Regular Graph and iii) Induced Subgraph. Give one example for each. **(06 Marks)**

b. For a graph with n vertices and m edges, if δ is minimum and Δ is maximum of the degrees of vertices show that $\delta \le (2m/n) \le \Delta$. **(05 Marks)**

c. Show that the following graph is isomorphic to Kuratowski's second Graph ($K_{3.3}$) **(05 Marks)**

Fig Q1(c)



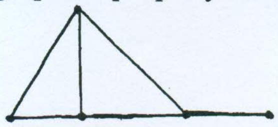
d. Write a note on Konigsberg's seven Bridge problems. **(04 Marks)**

2 a. Prove that a connected planar graph G with n vertices and m edges has exactly $(m - n + 2)$ regions in every one of its diagrams. **(07 Marks)**

b. State and explain Kuratowski's theorem. Show that the graphs K_5 and $K_{3.3}$ are non-planar by re-drawing them. **(06 Marks)**

c. Find the chromatic polynomial for the following graph. If 5 colors are available, in how many ways can the vertices of this graph be properly colored? **(07 Marks)**

Fig Q2(c)



3 a. Define Trees. Prove that a tree with n – vertices has n-1 edges. **(06 Marks)**

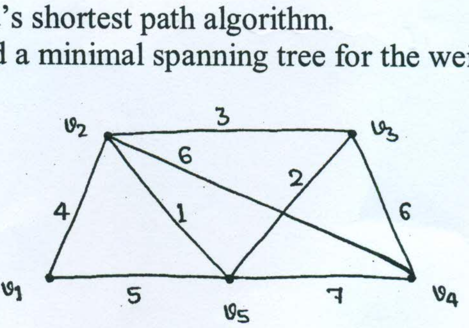
b. Define: i) Spanning Tree ii) Rooted Tree and iii) Full Binary Tree. Give one example for each. **(06 Marks)**

c. Explain prefix codes. Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. **(08 Marks)**

4 a. Explain the steps in Dijkstra's shortest path algorithm. **(06 Marks)**

b. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below. **(07 Marks)**

Fig Q4(b)



c. Three boys B_1, B_2, B_3 and four girls G_1, G_2, G_3, G_4 are such that i) B_1 is a cousin of G_1, G_3, G_4 ii) B_2 is a cousin of G_2 and G_4 iii) B_3 is a cousin of G_2 and G_3. If boys must marry a cousin girl, find the possible sets of such couples. **(07 Marks)**

PART – B

5 a. i) How many arrangements are there for all letters in the word SOCIOLOGICAL?
ii) In how many of these arrangements
- A and G are adjacent?
- All the vowels are adjacent? **(06 Marks)**

b. Find the co-efficient of
i) x^9y^3 in the expansion of $(2x-3y)^{12}$.
ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$ **(06 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that i) no container is left empty? ii) the fourth container gets an odd number of balls? **(08 Marks)**

6 a. State and prove the principle of Inclusion – Exclusion for n sets. **(05 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PIN, and BYTE occurs? **(05 Marks)**

c. In how many ways can the integers 1, 2, 3, ----- 10 be arranged in a line so that no even integer is in its natural place? **(05 Marks)**

d. An apple, a banana, a mango, and an orange are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango, and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(05 Marks)**

7 a. Using the generating functions, find the number of i) non negative, and ii) positive, integer solutions of the equation $x_1+x_2+x_3+x_4=25$. **(06 Marks)**

b. A bag contains a large number of red, green, white and black marbles, with atleast 24 of each colour. In how many ways can one select 24 of these marbles, so that there are even number of white marbles and atleast six black marbles. **(07 Marks)**

c. Using exponential generating function, find the number of ways in which four of the letters in the word ENGINE be arranged. **(07 Marks)**

8 a. Solve the recurrence relation.
$a_n = 2a_{n/2} + (n-1)$ for $n = 2^k, k \geq 1$, given $a_1 = 0$. **(08 Marks)**

b. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Using a recurrence relation determine the number of virus affected files in the system after one day. **(05 Marks)**

c. Find and solve a recurrence relation for the number of binary sequences of length $n \geq 1$ that have no consecutive O's. **(07 Marks)**

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